A comparison of ductile damage models with application to LCF

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Outline

- Motivation
- Comparison of models
- Numerical demonstrations
- Outlook
Motivation - Fatigue

- Fatigue, one major cause of failure

1988
Aloha Airline
Fatigue Failure

- for mechanical objects subjected to repeated loading.
Motivation - Fatigue

- Fatigue, one major cause of failure

- and also for human beings!
Motivation - LCF

- Low Cycle Fatigue can be observed when structures are subjected to **heavy load reversals** which leads to plastic strains and damage.

- Low cycle fatigue is important to be considered in the **design optimization** of industrial products, especially if failure occurs after relatively few cycles.
  - e.g. for airplanes, trains, cars or pressure vessels.
Motivation – Ductile Fracture

- Ductile fracture is a special cause of failure especially in plastic zones where the stress triaxiality is high.

- Ductile fracture is caused by the evolution of microvoids.

Ductile fracture surface of a round notched bar with Al 2024T351 material at 30 cycles (notch radius 2mm)
A simple ductile fracture model

- The famous Gurson model (Gurson 1977) looks like:

\[
F = \frac{\Sigma_{eq}^2}{\Sigma^2} + 2 f q_A \cosh \left( q_B \frac{3}{2} \frac{\Sigma_m}{\Sigma} \right) - 1 - q_C f^2
\]

\[
\Sigma_{eq}^2 = \frac{3}{2} \text{dev}(\sigma) : \text{dev}(\sigma) : \text{J}_2\text{-invariant}
\]

\[
\Sigma_m = \frac{1}{3} \text{tr}(\sigma) \quad : \text{mean stress}
\]

\[
\Sigma = \sigma_{Y_0} \quad : \text{ultimate yield limit}
\]

\[
f \quad : \text{microvoid volume fraction}
\]

\[
q_A, q_B, q_C \quad : \text{some fitting parameter}
\]
A simple ductile fracture model

- The famous Gurson model (Gurson 1977) looks like:

\[ F = \frac{\Sigma_{eq}^2}{\Sigma^2} + 2f^2 qA \cosh \left( qB \frac{3}{2} \frac{\Sigma m}{\Sigma} \right) - 1 - qC \, f^2 \]

- Void evolution (Chu & Needleman 1980):

\[ \dot{f} = \dot{f}_{\text{nucleation}} + \dot{f}_{\text{growth}} \]

\[ \dot{f}_{\text{nucleation}} = \frac{f_N}{s \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\epsilon_{eq}-e_N}{s} \right)^2 \right] \cdot c_{eq} \]

\[ \dot{f}_{\text{growth}} = (1 - f) \Delta \gamma \text{tr}(n) \]
The famous Gurson model (Gurson 1977) looks like:

\[ F = \frac{\Sigma_{eq}^2}{\Sigma^2} + 2 f q_A \cosh \left( q_B \frac{3}{2} \frac{\Sigma_m}{\Sigma} \right) - 1 - q_C f^2 \]

Questions:

- How can the Gurson model be used for repeated loading where the yield surface expands/moves in stress space?

- How can this be done in a micromechanically consistent way?
The famous Gurson model (Gurson 1977) looks like:

\[ F = \frac{\Sigma^2}{\Sigma^2} + 2f q_A \cosh \left( q_B \frac{3}{2} \frac{\Sigma^m}{\Sigma} \right) - 1 - q_C f^2 \]

The extension to isotropic hardening was accomplished by Gurson in the following way:

\[ (1 - f) \Sigma \dot{E} = \sigma : d^p \Rightarrow \Sigma = \Sigma(\bar{E}) \]

This represents some sort of a simple homogenization scheme!
The famous Gurson model (Gurson 1977) looks like:

\[ F = \frac{\Sigma_{eq}^2}{\Sigma^2} + 2 f q_A \cosh \left( q_B \frac{3}{2} \frac{\Sigma_m}{\Sigma} \right) - 1 - q_C f^2 \]

The extension to isotropic hardening was accomplished by Gurson in the following way:

\[(1 - f) \bar{\Sigma} \dot{\bar{E}} = \sigma : d^p \Rightarrow \bar{\Sigma} = \bar{\Sigma}(\bar{E})\]

It can be shown that \( \bar{F} \) is simply volume weighed over the volume cell:

\[ \bar{F} = \langle \epsilon_{eq}^p \rangle = \frac{\int_{a^3}^{b^3} \epsilon_{eq}^p(r) \, dr^3}{\int_{a^3}^{b^3} dr^3} \]

\[ f = \frac{a^3}{b^3} \]
The DLP-yield function for ductile fracture reads as:

\[ F = \frac{(\Sigma - A)^2_{eq}}{\Sigma^2_1} + 2f q \cosh \left( \frac{3}{2} \frac{(\Sigma - A)_m}{\Sigma_2} \right) - 1 - (qf)^2 \]

(Leblond et al. 1995) presented a more sophisticated model by considering the RVE in a sound way.

They found out that the yield limits \( \Sigma_1 \) and \( \Sigma_2 \) should be different and should be homogenized by different rules!
The Devaut-Leblond-Perrin model

- The DLP-yield function for ductile fracture reads as:

\[
F = \frac{(\Sigma - \Lambda)^2_{eq}}{\Sigma^2_1} + 2f q \cosh \left( \frac{3}{2} \frac{(\Sigma - \Lambda)_m}{\Sigma_2} \right) - 1 - (q f)^2
\]

(Leblond et al. 1995) presented a more sophisticated model by considering the RVE in a sound way.

Also, they presented a sound extension to kinematic hardening!
The Gurson model

The famous Gurson model (Gurson 1977) looks like:

\[ F = \frac{\Sigma_{eq}^2}{\Sigma^2} + 2 f q_A \cosh \left( q_B \frac{3}{2} \frac{\Sigma_m}{\Sigma} \right) - 1 - q_C f^2 \]

Some background:

The original Gurson model overpredicted the evolution of microvoids in monotonic tests and should not be able to predict any ratcheting in (“fictitious”) cyclic tests since the yield function depends only on a single yield limit.
The Devaut-Leblond-Perrin model

- The DLP-yield function for ductile fracture reads as:

\[
F = \frac{(\Sigma - A)^2_{eq}}{\Sigma^2_1} + 2fq \cosh \left( \frac{3}{2} \frac{(\Sigma - A)_m}{\Sigma_2} \right) - 1 - (qf)^2
\]

Instead, the DLP-model is able to predict increasing damage from cycle to cycle and reduces the overprediction of void growth for the monotonic loading case.
The Devaut-Leblond-Perrin model

- The DLP-yield function for ductile fracture reads as:

\[
F = \frac{(\Sigma - A)_{eq}^2}{\Sigma_1^2} + 2 f q \cosh \left( \frac{3}{2} \frac{(\Sigma - A)_m}{\Sigma_2} \right) - 1 - (q f)^2
\]

However, the homogenization procedure is somewhat different and more complicated:

\[
\Sigma_1 = \int_{a^3}^{b^3} \sigma(\langle c_{eq}^p \rangle_r) \, dr^3
\]
\[
\Sigma_2 = \int_{a^3}^{b^3} \sigma(\langle c_{eq}^p \rangle_r) \frac{dr^3}{r^3}
\]

which requires numerical integration schemes.
The DLP-yield function for ductile fracture reads as:

\[
F = \frac{(\Sigma - \Lambda)^2_{eq}}{\Sigma^2_1} + 2 f q \cosh \left( \frac{3}{2} \frac{(\Sigma - \Lambda)_m}{\Sigma_2} \right) - 1 - (q f)^2
\]

To simplify, (Leblond et al. 1995) considered only the proportional loading case and presented only a linear Prager rule for kinematic hardening.
The Devaut-Leblond-Perrin model

The DLP-yield function for ductile fracture reads as:

\[ F = \frac{(\Sigma - \Lambda)^2}{\Sigma_1^2} + 2f q \cosh \left(\frac{3}{2} \frac{(\Sigma - \Lambda)_m}{\Sigma_2}\right) - 1 - (qf)^2 \]

Questions:
- How can the model be extended to consider non-proportional loading?
- How can a more sophisticated kinematic hardening rule be implemented?
- How looks a reasonable extension to the large strain case?
- How can a coalescence model be implemented correctly?
The modified model – large strains

The DLP-yield condition for ductile fracture reads as:

\[ F = \left( \frac{\tilde{G}(\hat{\Sigma}^\# - \hat{\kappa})}{\tau_1^2} \right)^2 + 2 f^* q \cosh \left( \frac{3 (\tilde{G}(\hat{\Sigma}^\# - \hat{\kappa}))_m}{2 \ln(f^*)} \right) - 1 - (q f^*) \]

- \( \tilde{G} \): plastic metric of the intermediate configuration
- \( \hat{\Sigma}^\# = \hat{C} \hat{S} \hat{G}^{-1} \): symmetric Mandel stress tensor
- \( \hat{\kappa} \): symmetric back stress tensor

using multiplicative elasto-plasticity: \( F = F^e F^p \)

e.g. \( \hat{C} = F^p_{\triangleright}(C) = F^e \langle g \rangle \) or \( \hat{S} = F^p_{\triangleright}(S) = F^e \langle \tau \rangle = F^e \langle J \sigma \rangle \)

But note: \( \tau_1 = J^p \Sigma_1 \) and \( \tau_2 = J^p \Sigma_2 \), \( J^e \) disregarded
The modified model – hardening terms

- The DLP-yield condition for ductile fracture reads as:

\[
F = \left( \frac{\tilde{G}(\bar{\Sigma} - \bar{\kappa})}{\tau_1^2} \right)_{eq}^2 + 2 f^* q \cosh \left( \frac{\ln(f^* q)}{\ln(f)} \right) \frac{3}{2} \left( \frac{\tilde{G}(\bar{\Sigma} - \bar{\kappa})}{\tau_2} \right)_m - 1 - (q f^*)
\]

- The evolution equations for isotropic hardening look like:

\[
\tau_1 = J^p \Sigma_1 = J^p \frac{\int_{a^3}^{b^3} \sigma( < \epsilon_{eq}^p >_r \, dr^3)}{1 - f_0}
\]

\[
\tau_2 = J^p \Sigma_2 = J^p \frac{\int_{a^3}^{b^3} \sigma( < \epsilon_{eq}^p >_r \, dr^3)}{-\ln(f)}
\]

\[
< \epsilon_{eq}^p >_r^{1/2} = < d_{eq}^p >_r^{1/2} = \sqrt{D_{eq}^p + 4 \frac{b^6}{r^6} D_m^p}
\]

\[
f = \frac{a^3}{b^3}
\]
The modified model – hardening terms

- The DLP-yield condition for ductile fracture reads as:
  \[ F = \frac{(\tilde{G}(\hat{\Sigma}^\# - \tilde{\kappa}))_\text{eq}}{\tau_1^2} + 2f^*q \cosh \left( \frac{\ln(f^*q)}{\ln(f)} \right) \frac{3}{2} \frac{(\tilde{G}(\hat{\Sigma}^\# - \tilde{\kappa}))_m}{\tau_2} - 1 - (qf^*) \]

- For the mean part of the back stress tensor we use:
  \[ \tilde{\Lambda}_{m,n+1} = \int_{a_3}^{b_3} (\tilde{\Lambda}_{m,n} + \Delta t \tilde{\Lambda}_m) \frac{dr^3}{r^3} \]
  \[ = -\ln(f)\tilde{\Lambda}_{m,n} + \frac{4}{3} \Delta t D_p^m \beta_3 H \frac{b^3 - a^3}{a^3} \]

- For the deviatoric part we use an Chaboche-(A.-F.)-law:
  \[ \text{dev}(\tilde{\Lambda}_{n+1})_i = \text{dev}(\tilde{\Lambda}_n)_i + \beta_4 \left( c_i \Delta t \text{dev}(\tilde{D}^{p\#}) - \Delta \gamma \tilde{b}_i \text{dev}(\tilde{\Lambda}_{n+1})_i \right) \]
  \[ \tilde{\kappa} = J_p \left( (1 - qf^*) \sum_{i=1}^{4} \text{dev}(\tilde{\Lambda})_i + \tilde{\Lambda}_m \tilde{G}^{-1} \right) \]
The modified model – non-proportional

- The DLP-yield condition for ductile fracture reads as:

\[
F = \frac{(\hat{G}(\hat{\Sigma} - \hat{\kappa}))_\text{eq}}{\tau_1^2} + 2 f^* q \cosh \left( \frac{\ln(f^* q)}{\ln(f)} \right) \frac{3}{2} \left( \frac{\hat{G}(\hat{\Sigma} - \hat{\kappa}))_\text{m}}{\tau_2} \right) - 1 - (q f^*)
\]

- To consider the non-proportional loading case, we use an incremental time integration scheme.

\[\text{Firstly, compute } \Sigma_1 \text{ and } \Sigma_2 \text{ in terms of the } n\text{-values } \bar{E}_1^n \text{ and } \bar{E}_2^n \text{ and the time increment } < \dot{\varepsilon}^p_{\text{eq}} > r.\]

\[\text{Then, compute the } n+1\text{-values } \bar{E}_{1n+1}, \bar{E}_{2n+1} \text{ considering the final values of } \Sigma_1 \text{ and } \Sigma_2 \text{ using Newton’s method.}\]
The modified model – coalescence model

- The DLP-yield condition for ductile fracture reads as:

\[
F = \frac{(\tilde{G}(\tilde{\Sigma}^\parallel - \tilde{\kappa}))^2}{\tau^2} + 2f^* q \cosh \left( \frac{\ln(f^*)}{\ln(f)} \right) + \frac{3}{2} \left( \frac{\tilde{G}(\tilde{\Sigma}^\parallel - \tilde{\kappa})}{\tau} \right)_m \right) - 1 - (q f^*)
\]

- To model the process of coalescence, a simple empirical approach is used:

\[
f^* = \begin{cases} 
  f & \text{for } f \leq f_c \\
  f_c + K (f - f_c) & \text{for } f > f_c
\end{cases}
\]

with \( K = \frac{f_u^* - f_c}{f_f - f_c} \) and \( f_u^* = \frac{1}{q} \)

\( f_c \): critical microvoid volume fraction

\( f_f \): microvoid volume fraction at fracture

(Needleman & Tvergaard 1984)
Quality of approximation (Chaboche vs. Armstr.-Frederick):
Example to demonstrate accumulative damage:

- **System:**
  - Loading history:
    - $\varepsilon_{\text{max}} = 10.0\%$
    - $\varepsilon_{\text{min}} = 0.0\%$

- **Material data:**
  - $E = 208000$ MPa,
  - $\nu = 0.3$,
  - $\sigma_{Y_0} = 300$ MPa

- **Graph:**
  - Void volume fraction $f$ vs. number of cycles $N$:
    - Modified DLP, $q = 1.0$
    - Modified DLP, $q = 2.0$
Outlook

- What did we learn?
  The model is capable of representing the major aspects of ductile microvoid damage and is currently one of the most advanced models for isotropic ductile fracture! The application to LCF is possible and very promising!

- What has to be done in the future?
  To investigate the numerical response further, future work will focus especially on the behavior in different loading situations. To validate, unit cell analyses have to be done.