Erratum
Modeling of elasto-plastic material behavior and ductile micropore damage of metallic materials at large deformations

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October 2007, Review June 2011

Page 120

Wrong:

$$\frac{\partial W}{\partial \mathbf{g}} = \frac{\partial W}{\partial \mathbf{b}} \bullet\!\circ \mathbb{B} + \sum_{j=1}^{m1} \frac{\partial W}{\partial \mathbf{a}_{1j}} \bullet\!\circ \mathbb{A}_{ij} + \sum_{i=1}^{m2} \frac{\partial W}{\partial \mathbf{a}_{2i}} \bullet\!\circ \mathbb{A}_{2i} + \sum_{k=1}^{m3} \frac{\partial W}{\partial \mathbf{a}_{3k}} \bullet\!\circ \mathbb{A}_{3k} \quad (6.156)$$

Correct:

$$\frac{\partial W}{\partial \mathbf{g}} = \frac{\partial W}{\partial \mathbf{b}} \bullet\!\circ \mathbb{B} + \sum_{j=1}^{m1} \frac{\partial W}{\partial \mathbf{a}_{1j}} \bullet\!\circ \mathbb{A}_{1j} + \sum_{i=1}^{m2} \frac{\partial W}{\partial \mathbf{a}_{2i}} \bullet\!\circ \mathbb{A}_{2i} + \sum_{k=1}^{m3} \frac{\partial W}{\partial \mathbf{a}_{3k}} \bullet\!\circ \mathbb{A}_{3k} \quad (6.156)$$

Page 122

Wrong:

Remark: *Like mentioned above, the symmetry of the corresponding tensors must not be considered in the differentiation procedure. Along similar lines, we obtain the desired solution for a supersymmetric tangent operator by means of a final double contraction with* $\mathbb{S}$ *from the right and with* $\mathbb{S}^T$ *from the left. This rule holds in general. Applying the chain rule, the symmetry (or skew-symmetry) of a tensor should not be considered earlier in the chain but the last element i.e. artificially in some sense, since otherwise the result were wrong. Furthermore, the above reasoning applies in general even if we replaced the scalar-valued covariant tensor function* W *by an arbitrary covariant tensor of higher order which has the same functional dependencies.*

Correct:

Remark: *Like mentioned above, the symmetry of the corresponding tensors must not be considered in the differentiation procedure. Along similar lines, we obtain the desired solution for a supersymmetric tangent operator by means of a final double contraction with* $\mathbb{S}$ *from the right and with* $\mathbb{S}^T$ *from the left. This rule holds in general. Applying the chain rule, the symmetry (or skew-symmetry) of a tensor should not be considered earlier in the chain but the last element i.e. artificially in some sense, since otherwise the result were wrong. The mentioned relations are only satisfied if* W *is a scalar-valued covariant tensor function.*

Page 176

Wrong:

The algorithm is particularly simple if the basis must not be transformed during the return map such that all equations can be treated by means of the usual rules of matrix algebra. Whereas, with respect to the reference placement, $\boldsymbol{G}_i$ represents such a fixed basis, we must, with respect to the intermediate placement, resort to a basis of the intermediate placement which is computed at the time instant t_n and must keep this basis fixed during the return map. Only afterwards, if all unknowns are determined, we make a relative push forward of all components. For this reason, it is possible at all to choose $\hat{\hat{\mathbf{C}}}_{n+1}$ as independent quantity. Like well-known, in the framework of a convective coordinate system, the components of $\mathbf{C}$ are equal to g_{ij}.

Correct:

The algorithm is particularly simple if the basis must not be transformed during the return map such that all equations can be treated by means of the usual rules of matrix algebra. Whereas, with respect to the reference placement, $\boldsymbol{G}_i$ represents such a fixed basis, we must, with respect to the intermediate placement, resort to a basis of the intermediate placement which is computed at the time instant t_n and must keep this basis fixed during the return map. Only afterwards, if all unknowns are determined, we transform the components from the old to the new basis. For this reason, it is possible at all to choose $\hat{\hat{\mathbf{C}}}_{n+1}$ as independent quantity. Like well-known, in the framework of a convective coordinate system, the components of $\mathbf{C}$ are equal to g_{ij}.

Page 196

Wrong:

$$
\begin{aligned}
\mathbb{C}^{ep} &= 2\,\frac{\partial \mathbf{S}}{\partial \mathbf{C}} \bullet\!\circ \frac{1}{2}\,\Delta\mathbf{C} = 2\,\frac{\partial(\mathbf{F}^{p\,-1}\hat{\mathbf{S}}\mathbf{F}^{p\,-*})}{\partial \mathbf{C}} \bullet\!\circ \frac{1}{2}\,\Delta\mathbf{C} \qquad (7.457)\\
&= 2\,((\underline{\mathbf{F}}^{p\,-1}_{,\mathbf{C}}\,\underline{\hat{\mathbf{S}}\mathbf{F}}^{p-*} + \underline{\mathbf{F}}^{p\,-1}\underline{\hat{\mathbf{S}}\mathbf{F}}^{p\,-*}_{,\mathbf{C}}) + \underline{\mathbf{F}}^{p\,-1}\left(\frac{\partial\underline{\hat{\mathbf{S}}}}{\partial\underline{\hat{\mathbf{C}}}}\right)\underline{\mathbf{F}}^{p\,-*} \bullet\!\circ \frac{\partial\underline{\hat{\mathbf{C}}}_{n+1}}{\partial \mathbf{C}}) \bullet\!\circ \frac{1}{2}\,\Delta\mathbf{C}\ ,
\end{aligned}
$$

Correct:

$$
\begin{aligned}
\mathbb{C}^{ep} &= 2\,\frac{\partial \mathbf{S}}{\partial \mathbf{C}} = 2\,\frac{\partial(\mathbf{F}^{p\,-1}\hat{\mathbf{S}}\mathbf{F}^{p\,-*})}{\partial \mathbf{C}} \qquad (7.457)\\
&= 2\,((\underline{\mathbf{F}}^{p\,-1}_{,\mathbf{C}}\,\underline{\hat{\mathbf{S}}\mathbf{F}}^{p-*} + \underline{\mathbf{F}}^{p\,-1}\underline{\hat{\mathbf{S}}\mathbf{F}}^{p\,-*}_{,\mathbf{C}}) + \underline{\mathbf{F}}^{p\,-1}\left(\frac{\partial\underline{\hat{\mathbf{S}}}}{\partial\underline{\hat{\mathbf{C}}}}\right)\underline{\mathbf{F}}^{p\,-*} \bullet\!\circ \frac{\partial\underline{\hat{\mathbf{C}}}_{n+1}}{\partial \mathbf{C}})\ ,
\end{aligned}
$$

Page 198

Wrong:

$$\left.\frac{\partial \mathbf{R}_1}{\partial \mathbf{C}}\right|_{\Delta\gamma,\underline{\hat{\mathbf{N}}}^\flat=const.} = -\frac{1}{2}\,(\exp(-\Delta\gamma\underline{\hat{\mathbf{N}}}^\flat\,\underline{\hat{\mathbf{G}}}^{-1})\mathbf{F}_n^{p\,-*}\otimes\mathbf{F}^{p\,-1}\exp(-\Delta\gamma\,\underline{\hat{\mathbf{G}}}^{-1}\underline{\hat{\mathbf{N}}}^\flat) \quad (7.480)$$
$$+\exp(-\Delta\gamma\underline{\hat{\mathbf{N}}}^\flat\,\underline{\hat{\mathbf{G}}}^{-1})\mathbf{F}_n^{p\,-*}\boxtimes\mathbf{F}^{p\,-1}\exp(-\Delta\gamma\,\underline{\hat{\mathbf{G}}}^{-1}\underline{\hat{\mathbf{N}}}^\flat))\bullet\circ\frac{\partial\bar{\mathbf{C}}}{\partial\mathbf{C}}\,,$$

Correct:

$$\left.\frac{\partial \mathbf{R}_1}{\partial \mathbf{C}}\right|_{\Delta\gamma,\underline{\hat{\mathbf{N}}}^\flat=const.} = -\frac{1}{2}\,(\exp(-\Delta\gamma\underline{\hat{\mathbf{N}}}^\flat\,\underline{\hat{\mathbf{G}}}^{-1})\mathbf{F}_n^{p\,-*}\otimes\mathbf{F}_n^{p\,-1}\exp(-\Delta\gamma\,\underline{\hat{\mathbf{G}}}^{-1}\underline{\hat{\mathbf{N}}}^\flat) \quad (7.480)$$
$$+\exp(-\Delta\gamma\underline{\hat{\mathbf{N}}}^\flat\,\underline{\hat{\mathbf{G}}}^{-1})\mathbf{F}_n^{p\,-*}\boxtimes\mathbf{F}_n^{p\,-1}\exp(-\Delta\gamma\,\underline{\hat{\mathbf{G}}}^{-1}\underline{\hat{\mathbf{N}}}^\flat))\bullet\circ\frac{\partial\bar{\mathbf{C}}}{\partial\mathbf{C}}\,,$$

Page 200

Wrong:

$$\frac{\partial \hat{\underline{\bar{\mathbf{C}}}}_{n+1}}{\partial \mathbf{C}}\Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} = \frac{1}{2}\,(\exp(-\Delta\gamma \hat{\underline{\mathbf{N}}}^\flat \hat{\underline{\mathbf{G}}}^{-1}) \mathbf{F}_n^{p\,-*} \otimes \mathbf{F}^{p\,-1} \exp(-\Delta\gamma\, \hat{\underline{\mathbf{G}}}^{-1} \hat{\underline{\mathbf{N}}}^\flat) \tag{7.505}$$
$$+\exp(-\Delta\gamma \hat{\underline{\mathbf{N}}}^\flat \hat{\underline{\mathbf{G}}}^{-1}) \mathbf{F}_n^{p\,-*} \boxtimes \mathbf{F}^{p\,-1} \exp(-\Delta\gamma\, \hat{\underline{\mathbf{G}}}^{-1} \hat{\underline{\mathbf{N}}}^\flat)) \bullet\!\circ \frac{\partial \bar{\mathbf{C}}}{\partial \mathbf{C}}$$

Correct:

$$\frac{\partial \hat{\underline{\bar{\mathbf{C}}}}_{n+1}}{\partial \mathbf{C}}\Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} = \frac{1}{2}\,(\exp(-\Delta\gamma \hat{\underline{\mathbf{N}}}^\flat \hat{\underline{\mathbf{G}}}^{-1}) \mathbf{F}_n^{p\,-*} \otimes \mathbf{F}_n^{p\,-1} \exp(-\Delta\gamma\, \hat{\underline{\mathbf{G}}}^{-1} \hat{\underline{\mathbf{N}}}^\flat) \tag{7.505}$$
$$+\exp(-\Delta\gamma \hat{\underline{\mathbf{N}}}^\flat \hat{\underline{\mathbf{G}}}^{-1}) \mathbf{F}_n^{p\,-*} \boxtimes \mathbf{F}_n^{p\,-1} \exp(-\Delta\gamma\, \hat{\underline{\mathbf{G}}}^{-1} \hat{\underline{\mathbf{N}}}^\flat)) \bullet\!\circ \frac{\partial \bar{\mathbf{C}}}{\partial \mathbf{C}}$$

Page 251

Wrong:

$$
\begin{aligned}
\frac{\partial F}{\partial \mathbf{C}} \quad = \quad & \frac{\partial F}{\partial(\hat{\underline{\boldsymbol{\Sigma}}}^\sharp - \hat{\underline{\boldsymbol{\kappa}}})} \bullet\circ \left(\frac{\partial \hat{\underline{\boldsymbol{\Sigma}}}^\sharp}{\partial \hat{\hat{\underline{\mathbf{C}}}}} \bullet\circ \left(\frac{\partial \hat{\hat{\underline{\mathbf{C}}}}}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat, J^e = const.} + \frac{\partial \hat{\hat{\underline{\mathbf{C}}}}}{\partial J^e} \times \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} \right) \right. \\
& \left. + \frac{\partial \hat{\underline{\boldsymbol{\Sigma}}}^\sharp}{\partial J^e} \times \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} \right) \\
& + \left(\frac{\partial F}{\partial \tau_1} \frac{\partial \tau_1}{\partial J^e} + \frac{\partial F}{\partial \tau_2} \frac{\partial \tau_2}{\partial J^e} - \frac{\partial F}{\partial(\hat{\underline{\boldsymbol{\Sigma}}}^\sharp - \hat{\underline{\boldsymbol{\kappa}}})} \bullet\circ \frac{\partial \hat{\underline{\boldsymbol{\kappa}}}}{\partial J^e} \right) \times \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.}, \qquad (8.158)
\end{aligned}
$$

Correct:

$$
\begin{aligned}
\frac{\partial F}{\partial \mathbf{C}} \quad = \quad & \frac{\partial F}{\partial(\hat{\underline{\boldsymbol{\Sigma}}}^\sharp - \hat{\underline{\boldsymbol{\kappa}}})} \bullet\circ \left(\frac{\partial \hat{\underline{\boldsymbol{\Sigma}}}^\sharp}{\partial \hat{\hat{\underline{\mathbf{C}}}}} \bullet\circ \left(\frac{\partial \hat{\hat{\underline{\mathbf{C}}}}}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat, J^e = const.} + \frac{\partial \hat{\hat{\underline{\mathbf{C}}}}}{\partial J^e} \times \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} \right) \right. \\
& \left. + \frac{\partial \hat{\underline{\boldsymbol{\Sigma}}}^\sharp}{\partial J^e} \times \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} \right) \\
& + \left(\frac{\partial F}{\partial \tau_1} \frac{\partial \tau_1}{\partial J^e} + \frac{\partial F}{\partial \tau_2} \frac{\partial \tau_2}{\partial J^e} - \frac{\partial F}{\partial(\hat{\underline{\boldsymbol{\Sigma}}}^\sharp - \hat{\underline{\boldsymbol{\kappa}}})} \bullet\circ \frac{\partial \hat{\underline{\boldsymbol{\kappa}}}}{\partial J^e} \right) \frac{\partial J^e}{\partial \mathbf{C}} \Big|_{\Delta\gamma, \hat{\underline{\mathbf{N}}}^\flat = const.} , \qquad (8.158)
\end{aligned}
$$

Review June 2011:

Page 39

Wrong:

The arbitrariness of the time rate is valid in general, also for tensor functions of higher order, which is demonstrated for a second-order-valued tensor function with, again, two argument tensors $\mathbf{c}$ and $\mathbf{d}$:

$$\begin{aligned}
\mathrm{L}_{\mathbf{a}}(\mathbf{F}(\mathbf{c},\mathbf{d})) &= \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{c}} \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{c}) + \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{d}} \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{d}) \\
&= \frac{\partial \mathbf{F}(\mathbf{b}^{\triangleleft}(\mathbf{c}),\mathbf{b}^{\triangleleft}(\mathbf{d}))}{\partial \mathbf{b}^{\triangleleft}(\mathbf{c})} \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{c})) + \frac{\partial \mathbf{F}(\mathbf{b}^{\triangleleft}(\mathbf{c}),\mathbf{b}^{\triangleleft}(\mathbf{d}))}{\partial \mathbf{b}^{\triangleleft}(\mathbf{d})} \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{d})) \\
&= \mathbf{b}^{\triangleleft}\left(\frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{c}}\right) \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{c})) + \mathbf{b}^{\triangleleft}\left(\frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{d}}\right) \bullet\!\circ \mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{d})) \\
&= \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{c}} \bullet\!\circ \mathbf{b}_{\triangleright}(\mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{c}))) + \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{d}} \bullet\!\circ \mathbf{b}_{\triangleright}(\mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{d}))) \\
&= \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{c}} \bullet\!\circ \mathrm{L}_{\mathbf{ba}}(\mathbf{c}) + \frac{\partial \mathbf{F}(\mathbf{c},\mathbf{d})}{\partial \mathbf{d}} \bullet\!\circ \mathrm{L}_{\mathbf{ba}}(\mathbf{d}) \\
&= \mathrm{L}_{\mathbf{ba}}(\mathbf{F}(\mathbf{c},\mathbf{d}))\ . \qquad (3.80)
\end{aligned}$$

For the choice $\mathbf{b} = \mathbf{a}^{-1}$ the typical material time rate is included.

Correct:

The arbitrariness of the time rate is valid in general, also for tensor functions of higher order, which is demonstrated for a second-order-valued tensor function with, again, two argument tensors $\mathbf{c}$ and $\mathbf{d}$:

$$\mathrm{L}_{\mathbf{a}}(\mathbf{b}^{\triangleleft}(\mathbf{F}(\mathbf{c},\mathbf{d}))) = \mathrm{L}_{\mathbf{a}}(\mathbf{F}(\mathbf{b}^{\triangleleft}(\mathbf{c}),\mathbf{b}^{\triangleleft}(\mathbf{d}))) = \mathbf{b}^{\triangleleft}(\mathrm{L}_{\mathbf{ba}}(\mathbf{F}(\mathbf{c},\mathbf{d})))\ . \qquad (3.80)$$

For the choice $\mathbf{b} = \mathbf{a}^{-1}$ the typical material time rate is included.

Page 298

Wrong:

$$W(\mathbf{F}_\lambda) = \det(\mathbf{F}_\lambda) = 2+\lambda-\lambda^2 < \lambda\, 2+(1-\lambda)\, 2 = 2\,, \quad \lambda \in [0,1] \tag{D.12}$$

Correct:

$$W(\mathbf{F}_\lambda) = \det(\mathbf{F}_\lambda) = 2+\lambda-\lambda^2 > \lambda\, 2+(1-\lambda)\, 2 = 2\,, \quad \lambda \in [0,1] \tag{D.12}$$